Sequential Test:

It is a method in which the sample size 'n' is not assigned but the value of each freshly chosen observation decide whether we accept or reject the hypothesis or the test should be continued. A test used in a sequential method is called sequential test.

Sequential Probability Ratio Test:

Obtain the likelihood function of the given p.d.f.Then likelihood function is estimated at H_0 *i.e* $L(H_0)$ and at H_A i.e. $L(H_A)$.

Then by definition of sequential probability ratio test

$$k_0 > \frac{L(H_A)}{L(H_0)} > K_1$$

After simplification

$$c_0(n) > \sum X_i > c_1(n)$$

$$\frac{L(H_A)}{L(H_0)} \geq k_0 \quad iff \quad \sum X_i \geq c_1(n)$$

So reject $H_0: \theta = \theta_0$

And $$\frac{L(H_A)}{L(H_0)} \leq K_1 \quad iff \quad \sum X_i \geq c_0(n).$$

Then accept H_0

OR

$$k_0 < \frac{L(H_A)}{L(H_0)} < K_1$$

After simplification

$$c_0(n) < \sum X < c_1(n)$$

$$\frac{L(H_0)}{L(H_A)} < K_0 \quad iff \quad \sum X > c_1(n)$$

Then we reject $H_0: \theta = \theta_0$

And $$\frac{L(H_0)}{L(H_A)} > K_1 \quad iff \quad \sum X > c_0(n)$$

Then accept $H_0: \theta = \theta_0$

Thus we continue to observe outcomes as long as $c_0 < \sum X < c_1(n)$

OR

$$c_0(n) > \sum X > c_1(n)$$

Question 1:

Use sequential probability ratio test to test $H_0: \theta = 0.02 \; vs \; H_A: \theta = 0.07$ in a Poisson distribution with mean θ.

Show that this statistic can be based upon the statistic $\sum X$?

Solution:

As $X \sim P(X;\theta)$

$$f(x) = \frac{e^{-\theta}\theta^{x}}{X!} \qquad\qquad x:0,1,2,3\ldots..,\infty$$

Taking the likelihood function

$$L(x) = \prod_{i=1}^{n} f(x)$$

$$= e^{-n\theta} \frac{\theta^{\Sigma x}}{\prod_{i=1}^{n} x!}$$

As

$H_0: \theta = 0.02$ then we get

$$L(0.02, n) = L(H_0) = e^{-0.02n} (0.020^{\Sigma x} \Big/ \prod_{i=1}^{n} x!$$

And $H_A: \theta = 0.07$

$$L(0.07, n) = L(H_A) = e^{-0.07n} (0.07)^{\Sigma x} \Big/ \prod_{i=1}^{n} x!$$

Then by definition of sequential probability ratio test:

$$k_0 < \frac{L(H_0)}{L(H_A)} < k_1$$

$$k_0 < \frac{e^{-0.02n} (0.02)^{\Sigma x} \Big/ \prod_{i=1}^{n} x!}{e^{-0.07n} (0.07)^{\Sigma x} \Big/ \prod_{i=1}^{n} x} < k_1$$

$$k_0 < \left(\frac{0.02}{0.07}\right)^{\Sigma x} e^{-0.02+0.07n} < k_1$$

$$k_0 < \left(\frac{0.02}{0.07}\right)^{\Sigma x} e^{0.05n} < k_1$$

$$k_0 < (0.29)^{\Sigma x} e^{0.05n} < k_1$$

$$\frac{k_0}{e^{0.05n}} < (0.29)^{\Sigma x} < \frac{k_1}{e^{0.05n}}$$

Taking log of base 0.29 0n b.s

$$\log_{0.29}\left(\frac{k_0}{e^{0.05n}}\right) < \sum x < \log_{0.29}\left(\frac{k_1}{e^{0.05n}}\right)$$

Let

$$c_0(n) = \log_{0.29}\left(\frac{k_o}{e^{0.05n}}\right)$$

$$c_1(n) = \log_{0.29}\left(\frac{k_1}{e^{0.05n}}\right)$$

Then we get

$$c_0(n) < \sum x < c_1(n)$$

Now

$$\frac{L(H_0)}{L(H_1)} < k_0$$

If $\sum x > c_1(n)$ then we reject H_0 at $\theta = 0.02$

And

$$\frac{L(H_0)}{L(H_A)} > k_1$$

If $\sum x < c_0(n)$ then we accept H_0 at $\theta = 0.02$

Thus we continue to observed as long as $c_0(n) < \sum x < c_1(n)$.

Q.2

Let 'X' have a p.d.f $f(x;\theta) = \theta^x(1-\theta)^{1-x}$ with x: 0, 1.Then test $H_o\!: \theta = 1/3$ vs $H_A\!: \theta = 2/3$ by using sequential probability ratio test.

Q.3:

Let 'X' have a p.d.f $f(x;\theta) = \theta^x(1-\theta)^{1-x}$ with x: 0, 1.Then test $H_o\!: \theta = 1/3$ vs $H_A\!: \theta = 2/3$ by using sequential probability ratio test.

Solution:

As the given p.d.f.

$f(x;\theta) = \theta^x (1-\theta)^{1-x}$ x:0,1

Taking likelihood function

$L(x) = \theta^{\Sigma x}(1-\theta)^{n-\Sigma x}$

As $H_0: \theta = 1/3$ then we get

$$L(H_0) = \left(\frac{1}{3}\right)^{\Sigma x}\left(1-\frac{1}{3}\right)^{n-\Sigma x}$$

$$= \left(\frac{1}{3}\right)^{\Sigma x}\left(\frac{2}{3}\right)^{n-\Sigma x}$$

And $H_A: \theta = 2/3$

$$L(H_A) = \left(\frac{2}{3}\right)^{\Sigma x}\left(\frac{1}{3}\right)^{n-\Sigma x}$$

By definition of sequential probability ratio test:

$$k_0 < \frac{L(H_0)}{L(H_A)} < k_1$$

$$k_0 < \frac{(1/3)^{\Sigma x}(2/3)^{n-\Sigma x}}{(2/3)^{\Sigma x}(1/3)^{n-\Sigma x}} < k_1$$

$$k_0 < \frac{(1/3)^{\Sigma x}2^{n-\Sigma x}(1/3)^{n-\Sigma x}}{(2)^{\Sigma x}(1/3)^{\Sigma x}(1/3)^{n-\Sigma x}} < k_1$$

$$k_0 < 2^{n-\Sigma x}2^{-\Sigma x} < k_1$$

$$k_0 < 2^{n-2\Sigma x} < k_1$$

Taking log of base ‘2’ on b.s

$$\log_2 k_0 < -2\sum x + n < \log_2 k_1$$

$$\log_2 k_0 - n < -2\sum x < \log_2 k_1 - n$$

$$-1/2[\log_2 k_0 - n] < \sum x < -1/2[\log_2 k_1 - n]$$

Let

$$c_0(n) = \frac{-1}{2}\left[\log_2 k_0 - n\right]$$

$c_1(n) = \frac{-1}{2}[\log_2 k_1 - n]$

Then we get

$c_0(n) < \sum x < c_1(n)$

Now

$\frac{L(H_0)}{L(H_A)} < k_0$ If $\sum x_i > c_1(n)$

So we reject $H_0: \theta = 1/3$

And

$\frac{L(H_0)}{L(H_A)} > k_1$ If $\sum x_i < c_0(n)$

Then we accept $H_0: \theta = 1/3$

Thus we continue to observed as long as $c_0(n) < \sum x < c_1(n).$

Q.3:

Let 'X' have a p.d.f $f(x;\theta) = \theta^x (1-\theta)^{1-x}$ with x: 0, 1.Then test $H_o: \theta = 1/3 \; vs \; H_A: \theta = 2/3$ by using sequential probability ratio test.

Solution:

As the given p.d.f.

$f(x) = \theta^x (1-\theta)^{1-x}$ x:0,1

Taking likelihood function

$L(x) = \theta \sum^x (1-\theta)^{n-\sum x}$

As $H_0: \theta = 1/3$

$L(H_0) = (1/3)^{\sum x} (2/3)^{n-\sum x}$

As $H_A: \theta = 2/3$

$L(H_A) = (2/3)^{\sum x} (1/3)^{n-\sum x}$

By the definition of sequential probability ratio test

$k_0 > \frac{L(H_A)}{L(H_0)} > k_1$

$k_0 > \frac{2^{\sum x}}{2^{n-\sum x}} > k_1$

$k_0 > 2^{\sum x - n + \sum x} > k_1$

Taking log of base '2' on b.s.

$$\log_2 k_0 > 2\sum x - n > \log_2 k_1$$

$$\log_2 k_0 + n > 2\sum x > \log_2 k_1 + n$$

$$\frac{1}{2}[\log_2 k_0 + n] > \sum x > \frac{1}{2}[\log_2 k_1 + n]$$

Let

$$c_0(n) = \frac{1}{2}[\log_2 k_0 + n]$$

$$c_1(n) = \frac{1}{2}[\log_2 k_1 + n]$$

Then we get

$$c_0(n) > \sum x > c_1(n)$$

Now

$$\frac{L(H_A)}{L(H_0)} \geq k_0 \qquad iff \sum x_i \leq c_1(n)$$

So we reject H_0 $at\, \theta = 1/3$

And

$$\frac{L(H_A)}{L(H_0)} \leq k_1 \qquad iff \sum x_i \geq c_0(n)$$

So we accept H_0 $at\, \theta = 1/3$

Thus we continue to observed as long as $c_0(n) > \sum x > c_1(n)$.